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Return-Based Style Analysis: An Applied Study on Pension Funds Offered by the AGROS Institute

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Abstract: *The implementation of the Real Plan in 1994 caused a significant transformation in the Brazilian economic scenario, directly influencing how citizens manage their assets and select strategies and financial instruments for efficient management of resources. In this context, private pension plans stood out as a relevant alternative for long-term financial planning and capital accumulation. Given this scenario, this study applies the Return-Based Style Analysis (RBSA) methodology to evaluate investment strategies, resource allocation, and returns of two pension plans – B and CD – offered by Agros, a social security institute linked to the Federal University of Viçosa (UFV), responsible for administering complementary pension plans and health care. The results indicate that the RBSA is an effective tool for validating the management style of the analyzed plans. However, it was found that most of the plans' returns were, on average, linked to investments indexed to the Selic rate, reducing the predominance of passive management. This characteristic was more prominent in the CD plan, while in plan B, it was slightly mitigated by movements between asset classes over time.*

Keywords: *Economics; Management; Investments; Parametric Constraints; Multiple Linear Regression.*

Introduction

The history of the national economy, more specifically in the 80s, shows that Brazilians had to adapt to times of insecurity caused by the devaluation of their work and money due to the country's high inflation rates (Pinheiro *et al*., 1999). Between 1986 and 1991, Brazil experienced a series of failed attempts at monetary stabilization. During the government of José Sarney, five plans failed: Plano Cruzado (Feb/1986), Plano Cruzado II (Nov/1986), Plano Bresser (Jun/1987), Plano Feijão com Arroz (Jan/1988) and Plano Verão (Jan/1988). During the government of Fernando Collor, three more plans were also unsuccessful: Plano Collor (Mar/1990), Plano Collor II (Jan/1991) and Plano Marcílio (May/1991). Stabilization only occurred with Plano Real, implemented in July 1994, during the government of Itamar Franco, which was decisive in controlling inflation and reducing the Selic rate (Franco, 2017).

In this sense, in a recent and more stable economy, free from high inflation rates and, consequently, high interest rates, Brazilians had to admit that to obtain pressured returns on their investments, it would be necessary to take risks. This demand for new investment alternatives promotes growth and the search for financial resource management services in the market, traditionally carried out by companies (public or private) or specialized assets (Le Sourd, 2007).

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One proof of this was the growth of the risk investment market, as evidenced by the increase in the number of Brazilian investment funds and the financial volume they manage. More specifically, the number of investment funds offered in Brazil grew by approximately 390.37%, jumping from 4.540 funds in 2002 to 22.263 funds in 2020, according to data from the Brazilian Association of Financial and Capital Market Entities (ANBIMA), which corresponds to a compound growth of approximately 9.22% per year. The net equity of the investment fund industry in Brazil grew by approximately 13.009%, jumping from 46 billion reais in 1994 to approximately 6.030 trillion reais in 2020, corresponding to a compound growth of approximately 31.11% per year. It is also worth noting that the net equity managed by investment funds in Brazil in 2020 was equivalent to 81.15% of the national Gross Domestic Product (GDP) of the same year (approximately 7.4 trillion reais).

Furthermore, a recent technological revolution in the Brazilian financial market simplified and democratized the investment process. Therefore, investors now have access to platforms (or brokers) that offer a range of investment funds, as well as historical information on the returns of each of these funds, on the professionals who make decisions in these companies and on the respective capital allocation strategies adopted by them. Investors can now seek specialized help who can offer investments that maximize the probability of success, that is, the probability of obtaining a higher return conditioned on their risk profile (Le Sourd, 2007).

However, even with all this new information available, there are still difficulties, especially for investors, when comparing the investment policies and strategies of a given fund, since this task involves understanding the various asset classes that comprise the fund's investment portfolio, the different types of investment strategies, administration and custody fees, performance fees charged by management, etc. of the invested fund.

This study evaluates the application of shareholders' financial resources over time in two pension plans offered by Agros, a social security institute at the Federal University of Viçosa. To this end, the Return-Based Style Analysis methodology focused on plans B and CD. Specifically, the study seeks to (i) verify whether the allocations reported in the public statements and regulatory documents are, in fact, agreements by the institute's managers and (ii) assess whether the returns of the plans are compatible with the rates charged to shareholders, through a comparison with the rates applied in passive funds with a similar profile.

Statistical model

Return-Based Style Analysis, or RBSA, is a statistical methodology proposed by Sharpe (1988, 1992), whose main objective is to estimate the average exposure, over a given period, of an investment fund to different asset classes. These asset classes being associated with the markets to which the fund may be exposed. The general statistical model of RBSA, defined by a linear model of K factors, is reported below in Equation 1.

$$R_t = \left[\beta_1 F_{t1} + \beta_2 F_{t2} + \cdots + \beta_K F_{tK}\right] + e_t \quad (1)$$

In which R_t is the return of the fund under analysis at the instant t, for t = 2, 3 ... T; F_{tk} is the return of the *k-th* risk factor (or asset class) at the instant t, for t = 2, 3, ..., T and k = 1, 2, ..., K; β_k are the parameters of the model or the sensitivity of the returns of the fund under analysis concerning the *k-th* risk factor, for k = 1, 2, ..., K and assuming constant values of all other K – 1 risk factors, and e_t

is the random and unobservable error of the model, for t = 2, 3, ..., T, which has zero mean and homogeneous variance (σ^2).

Furthermore, according to Sharpe (1992), the terms of model (1) defined within the brackets, represent the "return attributable to the style" of the fund. The remaining portion, defined outside the brackets and associated with the et component, incorporates the effect of random factors, which may be associated with: i) unknown variables not included in the model, ii) variables that could not be measured and iii) the manager's selection capacity, that is, his ability to choose assets, to time purchases and sales, to use different investment strategies, etc.

It is worth noting that the original RBSA model, defined by Sharpe (1988, 1992), does not include the intercept term. However, De Roon *et al*. (2004), Marques *et al*. (2012), Pizzinga *et al*. (2012), and Schutt and Caldeira (2016), present in their works a reparameterization of the model, which adds an independent or intercept term (α or Jensen's Alpha, as it is also known) in model (1), to try to separate and quantify the exclusive effect associated with the manager's selective ability and attributing to the et component the effect of the other random factors (unknown and unmeasurable).

Response or Dependent Variable

Regarding the response or dependent variable of the model (1), represented by the vector $R_t=\left(R_2,R_3,\ldots,R_T\right)'$according to Morettin and Toloi (2006), its definition is initially given as specified in Equation 2:

$$r_t=\frac{Y_t-Y_{t-1}}{Y_{t-1}}=\frac{Y_t}{Y_{t-1}}-1,t=2,3,\ldots,T \tag{2}$$

In this case, Y_t is the continuous random variable that represents the asset's price or the value of the share of a plan/fund at instant t. Note also that by (2), r_1 should be considered as zero or disregarded. Subsequently, a modification of the simple net return is used (r_t), defined in (2), to represent the response or dependent variable of the model. This modification consists of taking $r_t + 1$, what is known as gross return, and applying the natural logarithm to it, as shown in Equation 3:

$$R_t=\ln\left(r_t+1\right)=\ln\left(\frac{Y_t}{Y_{t-1}}\right),t=2,3,\ldots,T \tag{3}$$

Explanatory or Independent Variables

The risk factors or asset classes are the market reference indexes or benchmarks that cover the fund's investment universe under analysis. It is worth noting that the explanatory or independent variables of model (1), whose effects are considered fixed, are defined by the returns of these reference indexes, also obtained from equations (2) and (3) and represented by the vector $\boldsymbol{F} = (F_{t1}, F_{t2}, \ldots, F_{tk})$ such that $\boldsymbol{F_{tk}} = (F_{1k}, F_{2k}, \ldots, F_{tk})'$, for k = 1, 2, ..., K and t = 2, 3, ..., T.

According to Sharpe (1992), risk factors, or asset classes, must meet three conditions:

- Be collectively exhaustive, that is, be able to adequately represent the entire universe of markets to which the fund under analysis is exposed;

- Be mutually exclusive, that is, represent a different type of market from the entire universe of markets to which the fund under analysis is exposed;
- They do not present the correlation between themselves, which can be verified if their returns are not correlated, that is, if $Cov\left(F_{kt}, F_{st}\right)=0 \ \forall_{\substack{k\neq s=1,2,\ldots,K. \\ t=2,3,\ldots,T}}$

Parametric Constraints

Another particularity of the model proposed by Sharpe (1988, 1992) was established with the intention that the coefficients can be interpreted as average estimates or proxies of the average percentage of allocation of the fund in each of the asset classes over the analyzed period (Brow and Goetzman, 1997). Additionally, the model imposes a restriction that the sum of the parameteres or sensitivities (β) must be equal 1 or 100%, that is:

$$\sum_{k=1}^{K} \beta_k = 1{,}00 \tag{4}$$

The restriction presented in (4), or portfolio restriction, when applied together with the statistical model (1) defines a variation of the RBSA methodology known as semi-strong RBSA (De Roon *et al*., 2004). Additionally, in some analysis situations, it may be necessary to impose another type of restriction. In these cases, a parametric inequality restriction, associated with each of the elements of the parameter vector, as presented in (5):

$$\beta_k \geq 0, k=1,2,\ldots,K \tag{5}$$

This type of restriction is helpful in the analysis of investment funds whose regulations do not allow the manager to operate in a leveraged manner, that is, when the manager is not allowed to allocate, in any of the asset classes, more capital or financial resources than the fund has in net equity under management. In other words, for this type of fund, operations such as raising funds using other assets as collateral, short selling of shares (a situation in which a share is sold that one does not own), and other forms of leverage could estimate one of the parameters negative are not permitted.

In this type of analysis, when the inequality restriction presented in (5) is imposed on model (1), along with the equality restriction in (4), a variation of the RBSA methodology occurs, known as strong RBSA (De Roon *et al*., 2004). Consequently, the semi-strong RBSA, as previously presented, should only be used to analyze investment funds that are allowed to use leverage, according to their regulations. More specifically, De Roon *et al*. (2004) argue that imposing the inequality restriction in (5) unnecessarily, when it is not consistent with the actual investment policy of the analyzed fund, may lead to biased estimates of the parameters or sensitivities.

Parameter Estimation

Estimation of model parameters or sensitivities, $\beta=\left(\beta_1,\beta_2,\ldots,\beta_K\right)'$, when no restriction is imposed (which is also known as weak RBSA), can be performed via the Ordinary Least Squares (OLS) method. In this case, it is a conventional multiple linear regression model. When the equality

or portfolio restriction is imposed (semi-strong RBSA), the OLS method can still be used with a reparameterization of the model (Marques *et al*., 2012) or the Lagrange Multiplier method (Gross, 2012).

However, according to Sharpe (1992), when the equality (4) and inequality (5) constraints are imposed jointly (strong RBSA), estimating the model parameters will consist of solving a quadratic programming problem. Therefore, a computational method is used to minimize a quadratic objective function, which would be the sum of the squares of the random errors, subject to a set of linear constraints, which would be the constraints defined in (4) and (5). According to Luenberger (1986), in the case of RBSA, the optimal solution obtained will provide a local minimum, which will also be the global minimum, since the objective function (SQE) is convex.

An important detail is that the original RBSA model is considered static, that is, the vector of estimated coefficients remains fixed or constant over time ($\boldsymbol{\beta}_t = \boldsymbol{\beta}$, $t = 2, 3, \ldots, T$). So, the estimates represent only the average sensitivities of the fund's returns to the risk factors or asset classes over the evaluated period. For more details on recent variations in the RBSA, see the works of Dor *et al*. (2005), Holmes and Faff (2008), Weng and Truck (2011), Swinkels and Sluis (2006), Mamaysky *et al* . (2008), Marques *et al*. (2012), Schutt and Caldeira (2016), Campani and Brito (2018).

Finally, a point not addressed in the original papers by Sharpe (1988) and Sharpe (1992) concerns the application of confidence intervals to the actual values of sensitivities, (β), especially in the case of strong RBSA, when equality and inequality restrictions are present, which means that the sampling distribution of the estimators is not known. More details on this can be found in Lobosco and DiBartolomeo (1997), who derived an approximate confidence interval (via the Delta method) for the sensitivities (valid only for particular situations) when the actual value of the sensitivities is not zero or one (values that belong to the edge of the parameter space). Alternatively, Kim *et al*. (2000) and Ottem Bams (2000), proposed alternatives to derive asymptotic confidence intervals based on the Andrews approximation (1997a, 1999), which are valid regardless of the actual values of the sensitivities.

Quality of Fit

To analyze the quality of the model's fit or its explanatory power, Sharpe (1992) suggested the coefficient of determination or R^2, defined by:

$$R^2 = \frac{SQRegression}{SQ\,Total} = 1 - \frac{SQWaste}{SQ\,Total} = 1 - \frac{Var\left(e_t\right)}{Var\left(R_t\right)} \tag{6}$$

The R^2 indicates how much of the variance of returns is explained by the RBSA applied to the risk factors or the selected asset classes. Still, in Equation (6), the ratio $Var(e_t)/Var(R_t)$ represents the proportion of the variance of portfolio returns that the RBSA model does not explain.

More specifically, Sharpe (1992) interprets R^2 as the percentage of the variation in return that the fund's style can explain. In this sense, many people mistakenly attribute the difference in $1 - R^2$ or the percentage not explained by the RBSA to risk factors, the portion of variability explained exclusively by the manager's selectivity or skill. Therefore, a low R^2 would imply that the fund under analysis presents more active management, while a high R^2 implies the opposite (more passive management). However, although this is an important result, it may not always be true, since an erroneous model specification may influence the value of the R^2.

Material and Methods

Characteristics of Agricultural Pension Plans

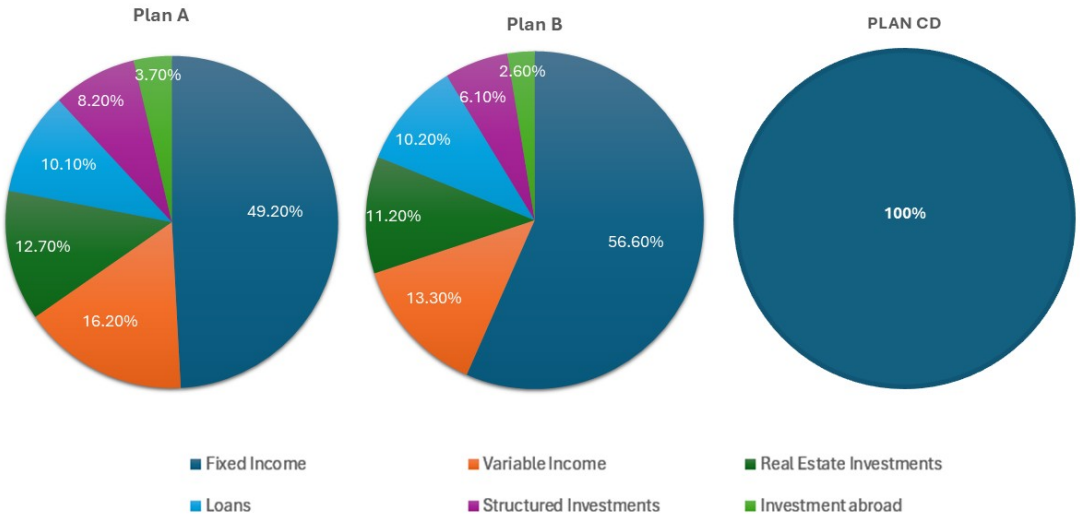
The UFV Institute of Social Security, Agros, is a closed entity of supplementary pensions (EFPC), created in May 1980 to supplement the benefits paid by social security to former employees of the Federal University of Viçosa (UFV). The institute offers three options of pension plans, called A, B and CD, registered, regulated and supervised by the National Superintendence of Supplementary Pensions (Previc), with the last two (B and CD) being the objects of study of this work. As explained in the following paragraphs, plan A was not analyzed because its composition is very similar to that of Plan B, which also has many shareholders and invested capital.

According to the 2020 Annual Information Report (RAI), the three pension plans together total 6,391 participants, with 4.52% belonging to plan A, 63.08% to plan B and , 32.40% to plan CD. The resources, or net equity that the plans have available to fulfill the pension benefits, reached R$ 1,327,933,352.80 in that same year, with 17.56% referring to plan A, 79.98% to plan B and 2.46% to plan CD.

Plans A and B, with defined benefits (a type of plan in which the participant retires with a pre-determined benefit), have a loading fee of 15% per year on the value of the current contribution. The CD plan, with defined contributions (a type of plan that allows the accumulation of contributions over time), in which the participant's future benefit will be a function of the accumulated balance, has an administrative fee of 0.8% per year on the current balance.

In addition, the main difference between these plans is associated with their respective investment policies. Plans A and B are similar and have greater diversification of financial capital allocation in different asset classes, unlike the CD plan, which only has fixed income as a segment or investment asset class. Figure 1 shows the average monthly allocation (based on the RAIs from 2012 to 2019) of resources by investment segment and illustrates the diversification of the plans.

Figure 1: Average monthly allocation (2012 to 2019) of plans A, B and CD.



Source: from the authors (Agros Institute: www.agros.org.br) (2025).

In short, concerning the investment segments presented in Figure 1, we have:

- Fixed Income: Investments made in Brazilian federal government bonds, debentures, financial bills and fixed-income funds;
- Variable Income: Investments in investment funds that buy shares in Brazilian companies;
- Real Estate Investments: Financial results obtained from transactions (purchase and sale) and rents received from properties located in the cities of Viçosa and Belo Horizonte;
- Loans and financing: Loan granting operations to pension plan participants;
- Structured Investments: Investments made in equity investment funds (FIP[1]) and multimarket funds;
- Investments Abroad: Applications made in investment funds that invest resources in fixed-income and variable-income in other countries.

Obtaining Data

The values associated with the response or dependent variable of the model (1) were obtained through direct contact with Agros, which provided the historical series of monthly returns of the pension funds offered by the institution. The monthly returns covering the period from January 2012 to December 2019 (96 observations for each series) were obtained. These returns are already free of fees.

As for the explanatory or independent variables of model (1), which are the risk factors or asset classes, these were chosen or selected based on the history of annual information reports for each pension plan, from 2012 to 2019, made publicly available by Agros. Additionally, the choice of explanatory variables also involved conversations with those responsible for the actuarial and investment sectors of the institute. Therefore, market indexes were sought that could best represent the asset classes associated with each fund's universe of investment possibilities. Such data were obtained through the Quantum Finance [2]platform, with each value present in the historical series representing the value of a daily closing price of the following indexes:

- IBOVESPA (main stock index on the Brazilian stock exchange);
- IFIX (main index of real estate funds on the Brazilian stock exchange);
- S&P 500 (Standard & Poor's 500 Index, which includes the 500 largest US companies listed on the NYSE and Nasdaq exchanges);
- PTAX DOLLAR (arithmetic average of the purchase and sale values of Dollars practiced by the leading exchange institutions in Brazil during the day);
- IMA-S (an index that replicates a portfolio of public securities whose remuneration or the percentage of interest to be received, upon maturity, is indexed to the SELIC rate);
- IMA-B 5 and IMA-B 5+ (an index that replicates a portfolio of government bonds maturing in less than 5 years and more than 5 years, respectively, whose remuneration or the percentage of interest to be received, upon maturity, is indexed to the IPCA);
- IMA-C (an index that replicates a portfolio of public securities whose remuneration or the percentage of interest to be received, upon maturity, is indexed to the IGP-M);

[1] The Equity Investment Fund (FIP) is a variable income investment made up of a portfolio of resources intended for investment in public, closed or limited companies, in the development phase.

[2] http://www.quantumfinance.com.br

- RF-M 1 and IRF-M 1+ (an index that replicates a portfolio of government bonds with maturity in less than 1 year and more than 1 year, respectively, whose remuneration or the percentage of interest to be received, upon maturity, is fixed).

It is worth noting that the daily historical series (associated with the variables mentioned above) was converted into a monthly series. To represent the observation of a given month, the value of the last business day of that month was used, with the aim that the periodicity of such a series would coincide with the periodicity of the return series provided by Agros. Subsequently, the returns of each of these indexes were calculated using formulas (2) and (3).

Computational Aspects

According to Agros' regulatory documents, none of the pension plans under study allow leverage, therefore, the RBSA statistical model in (1) must include both parametric constraints, equality in (4) and inequality in (5), which characterizes strong RBSA. This model was adjusted using the free software R (R Development Core Team, 2020), in which the quadprog package (TURLACH AND WEINGESSEL, 2013) and the solve.qp function will estimate the parameters by solving the quadratic programming problem.

Results and discussion

Agricultural Investment Funds

Table 1 describes the main statistics associated with the returns of the pension plans (B and CD) offered by Agros. Plan B obtained a higher average monthly return of 0.92%; however, it is associated with higher volatility (risk or standard deviation). Since the plan CD only has investments tied to fixed income assets, its lower profitability was already expected; however, its average value (0.81%) was only slightly lower than that of plan B, and with a much lower risk, as indicated by the standard deviation and also the Sharpe Ratio, which was approximately 3.06 times higher than that of plan B.

Table 1: Descriptive statistics of returns on funds B and CD.

Flat	N	Average Return	Standard Deviation	Sharpe Ratio[3]	Minimum Return	Maximum Return	Total Range of Returns
B	96	0.92%	1.01%	0.91	-2.09%	3.44%	5.53%
CD	96	0.81%	0.29%	2.79	0.14%	1.91%	2.05%

Source: from the authors (Agros Institute: www.agros.org.br) (2025).

It is also worth noting that the CD plan did not present a negative return in any month, unlike plan B. Corroborating this, the total amplitude of the return of plan B (5.53%) is greater than that of

[3] $IS_i = (R_i - R_f)/S_i$ that R_i is the average return of the i-th fund, S_i is the volatility or standard deviation of the i-th fund and R_f is the risk-free rate, which in this case, for convenience, will be assumed to be 0%. In general, the higher the Sharpe ratio, the higher the risk-weighted return.

plan CD (2.05%) since plan B has investments in other asset classes, such as variable income. This translates into greater volatility, caused by sudden movements (up and down) in the price of assets and, consequently, in the fund's return.

Market Indices

Twelve different market indexes covered the investment universe of the two plans analyzed in this study. Table 2 presents the statistical measures that describe the monthly returns of these indexes.

According to the descriptive statistics presented in Table 2, the highest average return observed was 1.78% and is associated with the S&P500 American stock index. One explanation for this is that after 2008, when the US experienced the height of the subprime financial crisis, the American economy recovered and began growing again, which caused the S&P500 index to begin a long-term upward trend, increasing its monthly returns during the period evaluated. Additionally, the same behavior was observed for investments made in the country's currency (dollar), which was appreciated against the real (almost 2 times in the period evaluated, dollar on 12/31/2008 = 2.34 reais and dollar on 12/31/2019 = 4.03 reais) and presented an average return of 0.79%.

Still in the economic context, a similar explanation can be attributed to the average return of the Bovespa index, which was 0.74%, the lowest recorded between 2012 and 2016. There was a downward trend in its prices, which intensified in mid-2014, when Brazil experienced a severe economic crisis. This behavior meant that for much of the period analyzed, the returns of this index were minimal or even negative. Note also that, according to the Sharpe index, the risk-weighted return was almost 3 times higher when comparing the American (S&P500) and Brazilian (IBOV) stock indexes. This fact demonstrates that, in addition to an opposite behavior in part of the period evaluated, the American market is less volatile since it is more developed, more efficient, and less impacted by a noisy financial environment than the Brazilian market financial environment.

The second lowest average monthly return observed (0.77%) is associated with the fixed income index IMA-S, which can be explained by the fact that the return of this index is linked to the return of the economy's basic interest rate or SELIC rate, which is considered by many to be the country's risk-free investment and, consequently, has the lowest expected return. However, mainly due to a cycle of rising interest rates that occurred between 2012 and 2016, partly caused by the effects of the economic crisis that the country experienced from 2014 onwards, the IMA-S obtained the highest Sharpe ratio (IS = 3.35) or the highest risk-weighted return, since there was a combination of higher profitability and lower volatility.

An interesting fact is that the fixed income indexes whose portfolio securities have short-term maturities (IMA-S and IRF-M1) had the highest risk-weighted return (IS = 3.35 and IS = 3.08, respectively). In addition to the cycle of rising interest rates, this can be explained by the fact that these indexes suffer less from the effect of the marking to market of securities (originated by the negotiation or trading of securities), which is the main factor causing volatility in the prices of these assets.

Table 2: Descriptive statistics of the monthly returns of the market indices used in constructing the models.

Index	N	Average Return	Standard Deviation	Sharpe Ratio	Minimum Return	Maximum Return	Total Range of Returns
IBOV	96	0.74%	5.74%	0.13	-12.62%	15.67%	28.29%
IFIX	96	1.05%	2.97%	0.35	-7.48%	10.10%	17.58%
S&P500	96	1.78%	4.81%	0.37	-14.61%	16.24%	30.85%
DÓLAR	96	0.79%	4.50%	0.18	-14.61%	13.14%	27.75%
IMA-S	96	0.77%	0.23%	3.35	0.38%	1.20%	1.58%
IMA-B 5	96	0.96%	0.83%	1.16	-1.57%	3.05%	4.62%
IMA-B 5+	96	1.16%	3.12%	0.37	-6.47%	10.13%	16.61%
IRF-M 1	96	0.80%	0.26%	3.08	0.20%	1.63%	1.83%
IRF-M 1+	96	1.01%	1.42%	0.71	-2.66%	5.08%	7.74%
IMA-C	96	1.08%	2.09%	0.52	-5.93%	6.31%	12.24%

Source: the authors (Quantum Finance: www.quantumfinance.com.br) (2025).

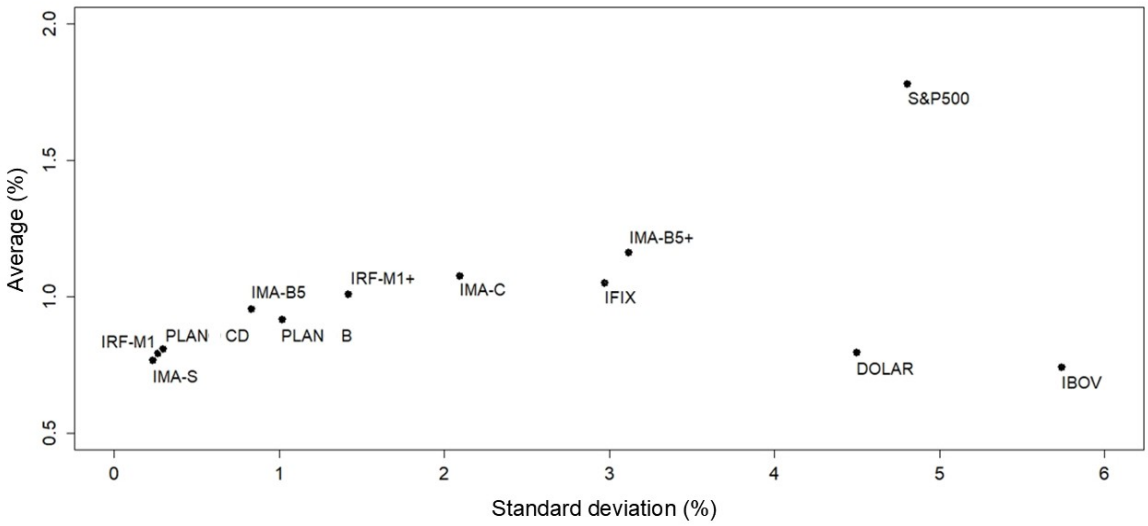
Finally, the average return of IFIX and IMA-C were very close, 1.05% and 1.08%, respectively. This may be associated with these two indexes having a direct and indirect relationship with the real estate market. IFIX is the index that represents the real estate fund market, real estate investments, and which was used in this study as a proxy to measure the appreciation of real estate, while the IMA-C is the index that has a portfolio of government bonds whose yields are linked to the IGP-M, which is also an index widely used to readjust real estate rents in Brazil. In other words, even though they represent different asset classes, the average returns may have been similar due to a particularity or an intersection between these two indexes, which involves the real estate marke.

Figure 2 presents a graph that illustrates the relationship between average returns and the risk or standard deviation of each asset class. These same results are also presented with pension plans B and CD offered by Agros.

According to Figure 2, the CD plan, whose investment universe involves only fixed income, obtained a risk-return ratio similar to that of the IMA-S and IRF-M1 indexes, which are indexes whose portfolio securities have short-term maturities. Since pension plans are long-term investments, it could be expected that the returns of the CD plan would be closer to the returns of fixed-income indexes that have securities with long-term maturities and that are even linked to inflation indexes, such as the IPCA and the IGP-M, in order to ensure that the purchasing power of the shareholder is not destroyed over time. However, this was not verified, possibly because these indexes have the lowest risk-weighted return (IS = 0.37 and IS = 0.52, respectively). According to Santos and Viana (2019), investments have been predominantly directed toward short-term assets, which is also observed in the

studies by Costa *et al*. (2000), Santos *et al*. (2004), and Campani and Brito (2018). This pattern is influenced by the country's history of high interest rates, which allows managers to obtain high returns with low risk through passive management.

Figure 2: Scatter plot of the averages (%) and standard deviations (%) of the monthly returns of the indexes and pension plans used in the construction of the models.



Source: from the authors (2025).

Plan B, which offers a broader and more diversified portfolio, but with the majority also linked to fixed-income investments (historically, on average, almost 60%), presented an average monthly return that slightly exceeded that of the IMA-S and IRF-M1 indexes, however, with greater risk involved. This result is curious and negative, in addition to diverging from the finance literature, as it does not seem to be interesting to include other asset classes in a portfolio (increasing the work involved in monitoring it) without verifying a risk reduction, since, according to Maestri and Malaquias (2016), this is the primary purpose of diversification, minimizing systematic risk.

When compared to the Bovespa index, plan B presented a higher risk-weighted return (IS = 0.13 and IS = 0.91, respectively), which can be explained by the fact that, during the period evaluated, changes were made in resources between the variable income and fixed income segments, as observed in 2014 and 2016, in which the percentage invested in variable income was reduced from 20.55% to 4.67% and that in fixed income was increased from 47.74% to 62.90%, aiming to position plan B, to try to obtain better returns through the cycle of high interest rates and to be less impacted by the collapse of the Bovespa index, which is interesting, as it seems to indicate that plan B has characteristics of a slightly more active (or less passive) management than that of plan CD.

Style Analysis

Regarding the style analysis applied to equity funds, Dibartolomeo and Witkowski (1997) used RBSA to classify several investment funds according to their estimated style and to verify whether the classification obtained was in accordance with their theoretical style, established a priori in the funds' regulations. To this end, the authors used data from American equity funds and concluded that of the 748 funds examined, 298 or approximately 39.83% of them presented returns

much more associated with other investment categories than with the categories defined a priori in their prospectuses.

Gallo and Lockwood (1999) used RBSA to analyze the effect of management changes on the style of investment funds. For this purpose, 69 American funds were evaluated, and it was found that approximately 65.2% of these funds presented style changes after the change in management or after the change of people responsible for the allocation strategies and decision-making in the fund.

Regarding RBSA applied to multimarket funds, Yoshinaga *et al*. (2009) analyzed Brazilian funds' style or allocation profile classified as leveraged and non-leveraged. The authors concluded that few use this strategy even when funds call themselves leveraged. This suggests that the fees charged on funds may not be justified, since managers do not use all permitted strategies.

Regarding the application of RBSA to private pension funds, Campani and Brito (2018) presented a study in which the performance of private pension funds was compared with the performance of passive or market reference funds, commonly referred to as benchmarks. Benchmarks are defined as a *priori* in the fund prospectus, and the main objective of managers is to exceed their returns, representing active management. In this sense, it was possible to conclude that several private pension funds charge high management fees, justifying them by active management, when in fact they present passive management, that is, often not exceeding or only obtaining returns similar to those of the benchmarks established for the fund.

RBSA applications in plan B

To corroborate the conclusions obtained above, Table 3 presents the results of applying strong RBSA to the data from plan B. The parameter estimates or estimated sensitivities for the risk factors or indices are reported. IBOV, IFIX, S&P500, DÓLAR, IMA-S, IMA-B, IMA-B 5+, IMA-C, IRF-M1 and IRF-M1+.

The parameter estimates indicated that, over the period evaluated, the average allocation of plan B resources was concentrated mainly in short-term fixed-income assets, whose return is linked to the Selic rate and pre-fixed interest rates. ($\hat{\beta}_5 = \hat{\beta}_{IMA-S} = 58,15\%$ e $\hat{\beta}_9 = \hat{\beta}_{IRF-M1} = 15.8\%$) and in variable income assets, whose return is linked to the return of shares in the index Bovespa ($\hat{\beta}_1 = \hat{\beta}_{IBOV} = 13.6\%$). These results align with those obtained by Santos and Viana (2019), who analyzed the monthly returns of 632 pension funds via RBSA, between October 2011 and April 2015, and found similar risk factors to be the most relevant. ($\hat{\beta}_{IMA-S} = 48.5\%$, $\hat{\beta}_{IBOV} = 13.8\%$ and $\hat{\beta}_{IRF} = 6.2\%$,where βIRF is the mean estimate associated with an index that combines the IRF-M1 e IRF-M1+).

Table 3: Results obtained for pension plan model B.

IBOV	IFIX	S&P	DÓLAR	IMA				IRF		%
				S	B5	B5+	C	M1	M1+	
$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	$\sum_{i=1}^{K} \hat{\beta}_i$
13.6%	0.0%	3.15%	0.0%	58.15%	3.0 %	0.39%	5.91%	15.8%	0.0 %	100.0%

$R^2 = 59.86\%$

Source: from the authors (2025).

Another result of Santos and Viana's (2019) work is that the estimate obtained for the inflation index, $\hat{\beta}_{IMA-B}=1.4\%$, in which $\hat{\beta}_{IMA-B}$ is associated with an index that combines the IMA-B5 and IMAB-5+, is also close to the average of the estimates obtained for the IMA indices in this work, $\hat{\beta}_6=\hat{\beta}_{IMA-B5}=3.0\%$ and $\hat{\beta}_7=\hat{\beta}_{IMA-B5+¿=0.39\%¿}$, therefore, on average, 1.69%.

More specifically, corroborating the results presented above, Santos and Viana (2019) concluded that the composition of pension fund portfolios in Brazil is concentrated in very short-term securities, with few variable income assets and negligible participation linked to the dollar. This last statement, regarding the dollar, is also in line with the results obtained in this study, since $\hat{\beta}_4=\hat{\beta}_{DOLAR}=0\%$.

The estimated parameter related to the index IMA-C ($\hat{\beta}_8=\hat{\beta}_{IMA-C}=5.91\%$) demonstrated that, on average, the return attributed to a small portion of the plan's investments, made over time, is associated with the return on investments indexed to the IGP-M. These investments, in the case of plan B, may be related to: i) a portfolio of government bonds or other fixed-income investments whose remuneration is linked to this index or ii) the adjustments and appreciation of properties and the rents of these real estate assets that belong to the plan's portfolio, which are also directly linked to the IGP-M. However, as the $\hat{\beta}_2=\hat{\beta}_{IFIX}=0\%$, It is more likely that option i) is the main reason for this result, since the returns of the IFIX, an index used in this research as an approximation to measure the appreciation of real estate, were not associated with the return of plan B.

The overseas investment segment, which was here, for convenience, simplified to an investment segment exclusive to the US, whose market was represented by the S&P500 index, presented an estimate of 3.15% ($\hat{\beta}_3=\hat{\beta}_{S\wedge P500}=3.15\%$). Therefore, since this estimate was very close to the actual average value (2.60%), obtained from information from the RAIs from 2012 to 2019 (Figure 1), there are indications that this investment segment of plan B may be little explored, since the association of the return of plan B with the return of the S&P500 index produced an estimate that leaves no room for other assets, from other large economies, to be included and generating returns for the portfolio in question.

As shown in Table 3, the adjusted model for plan B presented a coefficient of determination of 59.86%. In other words, it was possible to explain approximately 59.86% of the variation in the monthly returns of this plan through regression on the risk factors chosen and represented by the market indexes, which, according to the results of Schutt and Caldeira (2016) and Weng and Truck (2011), suggests an indication of passive management. However, since the fund's document history showed that movements between asset classes were made over time, it would not be correct to generalize this conclusion, since this is a positive point and suggests that the institution's management, concerning plan B, is not entirely passive.

RBSA applications in the CD plan

For the CD plan model, risk factors or indexes were used as explanatory variables: IMA-S, IMA-B5, IMA-B5+, IMA-C, IRF-M1, and IRFM-1+. Table 4 presents the estimated sensitivities of each factor, obtained from applying strong RBSA to this plan's data.

The results showed that, throughout the period analyzed, the CD plan's investment portfolio was, on average, allocated to short-term government bonds, whose remuneration was indexed to the Selic rate. ($\hat{\beta}_1 = \hat{\beta}_{IMA-S} = 88.63\%$), which implies low diversification of this plan's investment portfolio in the fixed income segment, presenting little relevant diversity of indexes and bond maturities.

Table 4: Results obtained for the CD plan model.

IMA				IRF		%
S	B5	B5+	C	M1	M1+	
$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\sum_{i=1}^{K} \hat{\beta}_i$
88.63%	7.56 %	0.42 %	0.36 %	0.00 %	3.03 %	100.0%

$R^2 = 52.11\%$

Source: from the authors (2025).

It is important to mention that, according to Agros' documents, the fixed-income investments permitted under the CD plan involve investments in federal government bonds, debentures, financial bills and fixed-income funds. Therefore, the previous result suggests that to obtain a return close to or practically associated with the return of the Selic rate, Agros managers invested the financial

resources of the shareholders, over time, in the assets described above. This is a negative point since the shareholder had to pay a fee twice to obtain the return of a passive market index with a higher risk-weighted return (see the Sharpe ratio in Tables 1 and 2). More specifically, the first fee, indirect, is paid when Agros, to make such investments, pays a fee to the National Treasury or to the issuer of a fixed-income fund, and the second, direct, when Agros charges the shareholder an administration fee on the current balance (0.8% per year). However, note that it is possible to obtain Selic rate returns by investing directly in the Selic treasury bond, made available by the National Treasury, with a custody fee of 0.2% per year on the bonds acquired. Suppose Agros' initial investment is in a fixed-income fund. In that case, the problem intensifies, since, according to a study carried out at the request of Exame magazine (https://invest.exame.com/me/fundos-de-renda-fixa-como-ficam-as-taxas-de-administracao-se-a-selic-subir) in 2020, the average rate charged by 87 fixed income funds was 2.45% per year on the amount held by the investor, and in 2021, 1.61%. For the IMA-B5 index, the model presented an allocation percentage of 7.56% ($\hat{\beta}_2 = \hat{\beta}_{IMA\text{-}B5} = 7.56\%$), the second largest among the estimates obtained. Santos and Viana (2019), in an analysis that included 457 pension funds classified by ANBIMA as "fixed income pension funds" over the period from October 2011 to April 2015, also found that the most relevant investments were indexed mainly to very short-term fixed interest rates and to the Selic.

Finally, the results found for the IRF-M1+ index indicated investments made in public securities whose remuneration is fixed and whose maturities are over 1 year, with an estimated average percentage of 3,03% ($\hat{\beta}_6 = \hat{\beta}_{IRF\text{-}M1+} = 3.03\%$). Campani and Brito (2018) also observed little association between pension plans and fixed-rate government bonds, stating that the search for more active management in this type of plan (fixed income) needs to involve more active action with bonds of this nature, especially when observing good opportunities in times of high-interest rate levels (mark-to-market gains).

As for the R^2 of the model developed for the CD plan, a coefficient of determination of 52.11% was found, that is, the model was able to explain approximately 52.11% of the variation in the monthly returns of this plan through the returns of the chosen risk factors, represented by the market indexes. In this case, the result suggests passive management, mainly because the estimate associated with the IMA-S index indicated that almost 90% of the return of the plan's portfolio was linked to the return of this index or the Selic rate, with no apparent changes or alterations in it, throughout the period evaluated.

Furthermore, an interesting point that RBSA helped clarify is the type of allocation made in the CD plan within the fixed income segment. The only information available in the RAI documents is that the CD plan allocates 100% of its net assets under management to fixed income. Therefore, there is no description of what type of fixed income it would be, about the volatility and maturity of the securities or about indexes. The RBSA applied to the CD plan assisted in demonstrating this.

Conclusions

The RBSA proved to be an effective tool for the purpose of this study, which was to validate the style of the pension plans B and CD offered by Agros, that is, to compare the theoretical allocations (defined in the regulations) with the real allocations (established in practice). The results indicated that the real allocations were very close to those reported in the regulatory documents, which is a positive point, since it translates into increased shareholder trust in the management.

However, on average, most of the plans' returns were associated with returns on investments indexed to the Selic rate, which is a negative point, as it translates into a strong indication of passive management. In plan B, unlike plan CD, where this characteristic is more evident, this passivity was slightly attenuated by movements between asset classes in the portfolio over time.

Due to the history of high interest rates in Brazil, it is common to see this characteristic (passivity) or similar results to those found in Agros plans in other pension plans offered by other institutions. That said, it is important to highlight that the results showed that the shareholder could obtain similar or even slightly higher returns and with lower risk if he chose to directly purchase Selic treasury bonds, directly from the national treasury, paying a single custody fee, which is even lower than the fee charged by Agros' CD pension plan, which is the only plan available to new shareholders at the moment.

References

ANDREWS, D. W. K. Estimation When a Parameter Is on a Boundary. **Econometrica**, v. 67, n. 6, p. 1341–1383, 1999.

ANDREWS, D. W. K. Estimation When a Parameter Is on a Boundary: Theory and Application, Cowles Foundation Discussion Paper, 1997a.

BROWN, S. J.; GOETZMANN, W. N. Mutual fund styles. **Journal of financial Economics**, Elsevier, v. 43, n. 3, p. 373–399, 1997.

CAMPANI, C. H.; BRITO, L. M. Fundos de previdência privada: passividade a preços de fundos ativos. **Revista Contabilidade & Finanças**, Scielo Brasil, v. 29, n. 76, p. 148–163, 2018.

COSTA Jr, C. J.; da COSTA Jr, N. C. A.; LEAL, R. P.; LEMGRUBER; E. F.; MESCOLIN, A. Mercado de Capitais: análise empírica no Brasil. Editora Atlas, 2000.

DEROON, F. A.; NIJMAN, T. E.; TER HORST, J. R. Evaluating style analysis. **Journal of Empirical Finance**, Elsevier, v. 11, n. 1, p. 29–53, 2004.

DIBARTOLOMEO, D.; WITKOWSKI, E. Mutual fund misclassification: Evidence based on style analysis. **Financial Analysts Journal**, Taylor & Francis, v. 53, n. 5, p. 32–43, 1997.

LOBOSCO, A.; DIBARTOLOMEO, D. Approximating the Confidence Intervals for Sharpe Style Weights. **Financial Analysts Journal**, v. 53, n. 4, p. 80–85, 1997.

DOR, A. B.; JAGANNATHAN, R.; MEIER, I. Understanding mutual fund and hedge fund styles using return-based style analysis. **The world of hedge funds:** characteristics and analysis. p. 63–108, 2005.

FRANCO, G. H. B. **A moeda e a lei:** Uma história monetária brasileira, 1933-2013. Editora Schwarcz-Companhia das Letras, 2017.

GALLO, J. G.; LOCKWOOD, L. J. Fund management changes and equity style shifts. **Financial Analysts Journal**, v. 55, n. 5, p. 44–52, 1999.

GROSS, J. Linear regression. Springer Science & Business Media, 2012. v. 175.
HOLMES, K. A.; FAFF, R. W. Style Analysis customized benchmarks, and managed funds: new evidence. **Applied Financial Economics Letters**, v. 4, n. 4, p. 253–258, 2008.

KIM, T; STONE, D.; WHITE, H. Asymptotic and Bayesian Confidence Intervals for Sharpe Style Weights. University of California, 2000.

LE SOURD, V. Performance measurement for traditional investment. **Financial Analysts Journal**, v. 58, n. 4, p. 36–52, 2007.

LUENBERGER, D. G. Control of linear dynamic market systems. **Journal of** Economic Dynamics and Control, V. 10, n. 3, p. 339–351, 1986.

MAESTRI, C. O. N. M., MALAQUIAS, R. F. Exposição fatores de mercado de fundos de investimentos no Brasil. Revista de Contabilidade e Finanças, v. 28, p. 61–76, 2016.

MAMAYSKY, H.; SPIEGEL, M.; ZHANG, H. Estimating the dynamics of mutual fund alphas and betas. **The Review of Financial Studies,** v. 21, n. 1, p. 233–264, 2008.

MARQUES, R.; PIZZINGA, A.; VEREDA, L. Restricted kalman filter applied to dynamic style analysis of actuarial funds. **Applied Stochastic Models in Business and Industry**, v. 28, n. 6, p. 558–570, 2012.

MORETTIN, P. A.; TOLOI, C. **Análises de séries temporais**. 2 ed. São Paulo: Blucher, 2006.

OTTEN, R.; BAMS, D. Statistical Test for Return-Based Style Analysis. 2000.

PIZZINGA, A. *et al*. Análise dinâmica de estilo para fundos de investimentos brasileiros. **Revista Brasileira de Risco e Seguro**, v. 7, n. 14, p. 1–34, 2012.

R Development Core Team (2020). **R: A language and environment for statistical computing**. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org/.

SCHUTT, I. G.; CALDEIRA, J. F. Análise de estilo dinâmica de fundos multimercados: aplicação para o mercado brasileiro. **Análise Econômica**, v. 34, n. 65, 2016.

SHARPE, W. F. Determining a fund's effective asset mix. **Investment management review**, v. 2, n. 6, p. 59–69, 1988.

SHARPE, W. F. Asset allocation: Management style and performance measurement. **Journal of portfolio Management**, v. 18, n. 2, p. 7–19, 1992.

SANTOS, R.; VIANNA, P. Coordenação de prazos e eficiência previdenciária, **Revista Brasileira de Economia**, v. 73, p. 121–135, 2019.

SANTOS, C. E. G.; MATSUMOTO, A. S.; PINHEIRO, C. A. O. A performance dos fundos abertos de previdência privada. Encontro norte-nordeste de finanças, 2004.

SWINKELS, L.; DER SLUIS, P. J. Van. Return-based style analysis with time-varying exposures. **The European Journal of Finance**, v. 12, n. 6–7, p. 529–552, 2006.

TURLLACH, B. A., WEINGESSEL A. (2013). **Quadprog: Functions to Solve Quadratic Programing Problems**. R package version 1.5-5, URL http://CRAN.R-project.org/package=quadprog/.

WENG, H.; TRUCK, S. Style analysis and value-at-risk of asia-focused hedge funds. **Pacific-Basin Finance Journal**, v. 19, n. 5, p. 491–510, 2011.

YOSHINAGA, C. E; JUNIOR, F. H. F. C.; ODA; A. L.; LUCCHESI, E. P. Análise de estilo em fundos multimercados com e sem alavancagem no brasil. **Revista Eletrônica de Gestão**, v. 2, n. 1, p. 9–21, 2009.