ISSN: 2317-0840

Tower of Hanoi in Mathematics classes: contributions to teaching geometric progression

Vanessa N. Oliveira[1], Rafaela de O. Alves[1], Rafael Pablo F. Cardoso[1], Breno Aparecido S. dos Reis[1], Ronaldo André Lopes[2†], Andréa Cardoso[3], José Carlos de Souza Júnior[3]

[1] *Federal University of Alfenas; Institute of Exact Sciences; Student of the Mathematics Course; Alfenas – Minas Gerais, Brazil.*
[2] *Federal University of São Carlos; Postgraduate Program in Education (PPGE); Student of the Doctorate in Education and Professor of Basic Education (SEE/MG); São Carlos – São Paulo, Brazil.*
[3] *Federal University of Alfenas; Institute of Exact Sciences; Professor of the Mathematics Department; Alfenas – Minas Gerais, Brazil.*

Abstract: *The Tower of Hanoi is a manipulable material that can promote learning of mathematical content, including sequences and geometric progression (BRASIL, 2018). The objective of this study is to report the results of a teaching intervention carried out at E. E. Samuel Engel, a partner in the Institutional Teaching Initiation Scholarship Program (PIBID) Mathematics. In order to relate the game with geometric progressions, the scholarship holders carried out an investigative activity with high school students, exploring the movements of the disks and possible patterns. Through group challenges, students were encouraged to find solutions with different numbers of disks in the Tower of Hanoi. We observed that, in the five classes, students identified patterns in the movements necessary to solve the game through exploration. Furthermore, in groups, they determined the minimum number of movements and, in some cases, mathematically generalized the calculation for any number of disks, using formulas and algorithms. In general, students welcomed the investigative approach, considering it more interesting than conventional classes. However, some groups found it difficult to formalize their reasoning and directly associate the Tower of Hanoi with geometric progressions, being helped by PIBID scholarship holders. The intervention, therefore, enabled students to establish connections between the theory of geometric progressions and the game, contributing to the consolidation of the content. Therefore, this study highlights the importance of activities involving manipulable materials, games, and mathematical investigation as drivers of learning in high school.*

Keywords: *Tower of Hanoi; Games in Education; Mathematics Education; Mathematical Investigation.*

† **Corresponding author: ronaldo.lopes@sou.unifal-mg.edu.br**

Manuscript received: 18/11/2024; Revised: 28/12/2024; Accepted: 08/01/2025

Introduction

The Brazilian National Common Curriculum Base (BNCC - Base Nacional Comum Curricular) (Brasil, 2018) is a guiding document that defines the main learning to be developed in basic education, considering all cycles of education. Its objectives include promotion of equality, aiming at the students' holistic education seeking a fair and inclusive society.

Regarding high school, the BNCC proposes deepening the contents approached in elementary school, focusing on *mathematical maturity*. The document also provides for the integration of mathematics to the students' reality, thus favoring critical thinking and the application of mathematics in everyday situations.

According to the BNCC, Brazilian states elaborate their specific curricula, which cover local demands, such as the Minas Gerais Reference Curriculum (CRMG- Currículo Referência de Minas Gerais) (Minas Gerais, 2020), which is a curriculum based on other official documents and on the recognition and appreciation of peoples, cultures, and traditions existing in that state. Targeting the holistic education of students, those documents highlight that research must be developed in the school context involving knowledge already consolidated by the sciences, thus enabling experiences with diverse procedures and the development of investigation skills, model construction, and problem solving (Brasil, 2018).

Regarding high school competences, both BNCC and CRMG present in the Specific Competence number 5 the idea of investigating and establishing hypotheses about different concepts and mathematical properties, with resources and strategies that involve, for example, the observation of patterns, among other aspects.

In such context, the skill EM13MAT508 is emphasized. Its objective is to identify and associate numerical sequences – geometric progressions (GP) – to discrete domain exponential functions to analyze properties, including deduction of certain formulas and problem solving (Brasil, 2018, p.533).

Considering the school demand for the approach of this competence and skill in high school, an intervention was carried out involving geometric progressions, using the material called Tower of Hanoi, in a mathematical investigation with classes of the 2nd and 3rd years of high school in a public school. The central idea, according to the official documents, was to motivate students to reason, represent, argument, and validate their hypotheses regarding the theme approached, thus promoting concept learning and deduction of formulas to represent geometric progressions (Brasil, 2018, p. 519).

To approach the theme proposed, we considered mathematical investigation as the main trend, with the use of a landscape of investigation (Skovsmose, 2000). Such approach favors the discovery and exploration of materials and content by students, thus contributing directly to their understanding of the themes approached and becoming an alternative to the traditional mathematics lessons, placing students as the protagonists of the teaching and learning process.

Traditionally, mathematics has been taught using the exercise paradigm methodology, which involves the resolution of a set of exercises as the central learning process. In that method, students are exposed to several problems, with different levels of progressive complexity. Therefore, the teacher presents examples, with rules, and a kind of recipe to solve them. Next, students only reproduce what the teacher told them to do, passively, in an exhausting way. Such methodology might present some positive points such as content memorization; however, it should not be the only method used in the classroom, even if it still predominates in the classroom basic education (Skovsmose, 2014).

As an alternative to the exercise paradigm, Skovsmose (2000) proposes landscapes of investigation, which is a student-centered methodology. It promotes students' protagonism in the classroom, in an active way, where they are challenged to find patterns, formulate hypotheses, and share their ideas with the group. In addition, landscapes of investigation favor the construction of mathematical arguments, deduction of formulas, and the visualization of the content applicability in a real context, for example.

Taking that into consideration, the following question is raised: How can the use of the Tower of Hanoi to approach geometric progressions in mathematical investigation favor the understanding of this content in classes of the 2nd and 3rd years of high school in a public school? To shed light on this theme, the objective of this article is to report the results of a teaching intervention carried out at the Samuel Engel State School, a partner institution in the Institutional Teaching Initiation Scholarship Program (PIBID) Mathematics.

The PIBID program and mathematics lessons

The Institutional Teaching Initiation Scholarship Program (PIBID) is a vital initiative in the context of the Ministry of Education Teachers' Education National Policy, playing a fundamental role in the improvement of higher education for teachers and increase in the public basic education in Brazil. Its main aim is to promote teaching initiation, integrating higher education and basic education. The PIBID program contributes to the advancement of higher education for teachers and to the improvement of the Brazilian public education quality. The program aims to promote the insertion of teachers in the public-school routine in the first half of the teaching courses by granting scholarships to students, public school teachers, and professors working in higher education institutions (HEI). This synergy between theory and practice, provided by the PIBID, strengthens the education of future teachers and also impacts basic education positively by means of collaborative actions and enriching education experiences (CAPES, 2023).

At the Federal University of Alfenas (UNIFAL-MG), the PIBID unfolds into several subprojects that cover areas such as mathematics, language (Spanish and Portuguese), history, biological sciences, social sciences, geography, physics, education, and chemistry. Such subprojects focus on the improvement of the future teachers' education and also show a significant impact on the schools in Southern Minas Gerais. They are developed in municipalities such as Alfenas, Fama, Serrania, and Poço Fundo, among others. Specifically in the mathematics subproject at UNIFAL-MG, the work team comprises two area coordinators, two mathematics supervisor teachers, and sixteen teaching initiation (TI) scholarship holders, divided into two groups, each accompanying a supervisor who teaches in a partner school. The schools are Samuel Engel State School and Professor Levindo Lambert State School, both located in Alfenas-MG. Each group is responsible for planning and developing interventions, activities, and classroom observations. In this article, we report an experience developed in the partner school Samuel Engel State School, where the TI scholarship holders taught high school classes, that is, four classes in the 2nd year, one in the 3rd year, and one in the 1st year of the New High School, implemented in 2022 in Minas Gerais.

The partnership between UNIFAL and the schools is ruled by the Call 31/2022 published on 07/10/2022, which was in force up to March 2024. During that period, several collaborative actions were developed, including classroom observations and interactions between the students, schoolteachers, and professors from the higher education institutions. Not only did those actions benefit the school students, but also provided them with enriching educational experiences, which also enriched the education of the undergraduates, thus creating a solid bridge between theory and practice.

Considering the school students' perception regarding PIBID and the UNIFAL teachers' view of the program, it seems relevant to explore the classroom interaction, the occurrence of doubts, and the participation in diverse tasks. Next, we sought to offer a reflection on how both groups realized the benefits of the program for their learning process and teachers' education, resulting in invaluable perceptions of the PIBID real impact on basic education and in the professional development of future teachers.

Landscapes of investigation in mathematics lessons

According to Skovsmose (2000), the traditional mathematics education based on the exercise paradigm, is marked by the teacher's central role, as a presenter of ideas and mathematical techniques to solve problems related to a certain content. The next step is the presentation of some examples and, next, students solve several exercises, commonly extracted from textbooks or workbooks. In general, students accept the exercise data without questioning it and the information in the instructions are enough to solve the problem.

On the other hand, landscapes of investigation are part of a methodology in which students are invited to explore and argument about the content. They provide a space for dialogue and groupwork, in which hypotheses are raised about the theme proposed by the teacher. In these sceneries, students ask questions and seek explanation for certain results. The invitation might occur by asking questions such as "what might happen if ..."? and the students' acceptance of the invitation is demonstrated when students answer, for instance, "yes, this might happen if..." (Skovsmose, 2000, p. 6). For example, when using the Tower of Hanoi, a teacher might ask: "What happens if we move the tower disks to the left? Do you think we will need a lower number of movements?" Students accept this invitation when they try to manipulate the material to find out the number of movements, saying how many movements they did, or by giving some oral answer, such as: "yes, we can try it."

Clearly, in the classroom context, there is not a single correct or suitable methodology. Thus, it is not our intention to exhaust the methodological possibilities in this text, but rather to present a teaching intervention involving mathematical investigation that was successfully developed.

Considering that, Skovsmose (2000) emphasized the formation of six different learning environments, which involve the exercise paradigm and landscapes of investigation. Therefore, we considered the existence of three different types of references, with tasks that refer to pure mathematics, semi-reality, and real-life situations. See the learning environments below (Chart 1).

Chart 1: Learning environments

	Exercise paradigm	Landscape of investigation
References to pure mathematics	1	2
References to semi-reality	3	4
References to reality	5	6

Source: Skovsmose (2000).

The learning environments described present specific characteristics that can be exemplified as follows:

Learning environments 1 and 2 refer to pure mathematics. Environment 1, with the exercise paradigm activities includes exercises that apply concepts but no context. They are exemplified by activities such as algebraic operations, calculation, and any other type of activity that does not require anything else but pure mathematics. On the other hand, even having the same reference, the activities representing environment 2 also aim at concept application, but as an investigation, in which the application is not direct. As an example, we can mention the identification of patterns and properties in multiplication, such as commutativity in a Pythagorean Table, which is a different representation of the multiplication table, or even a task investigating the relations between the measures of the sides and angles of a right triangle using the Pythagorean Theorem and sine, cosine and tangent.

Environments 3 and 4 refer to semi-reality, which is characterized by a reality simulation. In environment 3, activities are contextualized in this “artificial reality”, but they do not aim at understanding and questioning such context, for example, in problems that present data that is not possible in reality, only to be identified and used by students, for instance, in the resolution of hypothetical problems, in which the data is not questioned. In landscapes of investigation in environment 4, the tasks are not real either, but they require investigation that leads students to question and understand the context, so that some space is created for discussions. Some examples of activities in this environment include problems that require the formulation of hypotheses, inferences, and semi-reality exploration by students.

Finally, environments 5 and 6 se refer to reality. Therefore, the activities in environment 5 present problem situations with real data, but such data is not discussed, and the main aim of the tasks is to apply concepts with real data such as statistical data from research agencies by sampling. However, the landscapes of investigation in environment 6 propose investigation tasks in real contexts, which can be carried out working with projects, which leads students to investigate. Unlike environment 5, one example of tasks in environment 6 is the use of statistical data observed and presented by students applying mathematical concepts planned by the teacher and others that might be necessary. Activities in this environment promote greater involvement of students and longer development time.

Methodology

In the presence of the PIBID-Mathematics at the Samuel Engel State School, the supervisor teacher presented the demand of approaching the geometric progression (GP) content with the high school 2nd and 3rd year classes. Therefore, a teaching intervention was organized involving the theme, considering the provisions of the BNCC (Brasil, 2018) and the Minas Gerais Reference Curriculum (Minas Gerais, 2020). The methodology adopted landscapes of investigation by Ole Skovsmose (2000), which involves the mathematical investigation trend.

The activity was developed by four scholarship holders in the Mathematics subproject of the Institutional Teaching Initiation Scholarship Program (PIBID) of the Federal University of Alfenas. The Samuel Engel State School, in Alfenas, Minas Gerais, is a partner in this project. The planning of the interventions to be carried out at the partner school was devised in meetings held by the supervisor teacher and the teaching initiation scholarship holders, who discussed the process of building up and developing the activities aiming at promoting their contact with teaching, favoring the application of tasks and reflection on the experiences lived at university in the school context.

Three 2nd year classes and one 3rd year class took part in the activity involving around 140 students. All classes had had some contact with arithmetic progressions, but not all classes knew geometric progressions (GP). Aiming at carrying out a dynamic activity, with greater interaction between students and the visualization of geometric progressions in practice, we opted for using the manipulable material called Tower of Hanoi.

The Tower of Hanoi is a manipulable material based on an Indian legend and created by the French mathematician Édouard Lucas in 1883. The legend supporting the game is that during the creation of the world there was a tray with three diamond needles fixed on it and God placed 64 disks made of pure gold in different sizes stacked in descending order with the larger disk on the bottom and the smaller disks on the top. After having placed the disks, the priests followed the Brahma immutable laws which established that the priest in charge could not move more than one disk at a time, and that the disk should be put on a different needle, so that the smaller disk was always on top of the larger one, never the other way out. When all 64 disks were transferred from the needle placed by God on the day of creation to the other needle, the world would come to an end (COSTA, 2010).

Therefore, based on the legend, the game rules are the same, that is, only one disk can be moved at time to the other rod, a larger one can never go up the smaller one and the game ends when all initial disks are around a different rod. The number of disks used might vary according to the objective proposed. Considering that, we used the manipulable material (Figure 1) to carry out the teaching intervention as follows:

Figure 1: Tower of Hanoi.

Source: from the authors (2025).

To plan the intervention, the scholarship holders created a lesson plan based on Costa (2010), in which that author reports his experience using the Tower of Hanoi at a school in the south of the country. In the lesson plan, the scholarship holders planned each part of the application, highlighting the main points and including objectives, skills, development, and appendices. The main objective of the intervention was to make students relate the least number of movements in the Tower of Hanoi with the idea of geometric progression, thus generalizing the result. At the time, a scholarship holder was responsible for taking notes during the intervention, including students' right moves, mistakes and comments for a specific analysis of the activity results.

The activity was planned and applied in a 50-minute lesson in each class, with the following development: the class was divided into groups of 4 and 5 students. Initially, students faced the challenge of moving 5 disks from one rod to the other. If they were not able to, the number was reduced to 4 disks, keeping the same challenge. After concluding the activity, the groups received some prizes prepared by the team. In that first moment, the objective was to allow students to observe how the proposed game and the Tower of Hanoi worked and whether there were strategies to complete the challenge, without worrying about the least number of movements. Next, they received the notebook (Figure 2) to take notes and help them to keep track of the number of movements, which was the challenge. At that point, students should record the results of their "minimum number of movements" and later on verify whether they were right or wrong. At the time, the scholarship holders provoked some questioning of the existence of a pattern in the results found, and if so, it should be recorded in the column "reasoning". Going beyond the material exploration carried out in the first phase, the second phase aimed at developing some mathematical investigation as proposed by Skovsmose (2000). Finally, students shared the results, and with the table completed, the scholarship holders presented a possible solution. If the students could not identify any pattern, they would show the general formula for the least number of movements, relating it to geometric progression (GP).

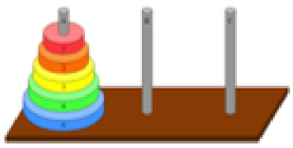
Figure 2: Notebook.

TOWER OF HANOI

History: The creation of the Tower of Hanoi was inspired by a Hindi legend, which described a temple in Benares, a holy city in India, where there was a Brahma sacred tower, whose function was to improve the mental discipline of young monks. According to the legend, in the great temple of Benares, under the dome that marked the center of the world, there is a brass plate on which three diamond poles are fixed. Around one of those poles, the Brahma god, at the time of the creation of the world, placed 64 disks made of pure gold in such a way that the largest disk was in contact with the brass plate and the others followed in descending order until reaching the top of the pole. The monks were told to transfer the tower formed by the disks from one rod to the other, using the third as an ancillary, but they could only move a disk at a time and never place a larger disk on top of a smaller one. The monks should work efficiently night and day until and when they finished the work, the temple would be turned into dust and the world would come to an end.

Source: https://www.ibilce.unesp.br/Home/Departamentos/Matematica/labmat/torre_de_hanoi.pdf

Pieces of the Game:
. 3 wooden rods fixed on a platform;
. 6 wooden disks of different sizes forming a tower;
Game rules:
. A larger disk cannot be put on top of a smaller one
. Only one disk can be moved at a time
. The objective is to place the whole tower at one end, either A or C.



To carry out the activity, use the table below to organize our investigations and find a possible formula for the minimum number of movements needed to finish the game.

Number of disks	Your number of movements	Least number of movements	Reasoning

Source: from the authors (2025).

Results and Discussions

The results obtained during the intervention revealed the students' positive perception of the investigation approach, which they found more attractive when compared to traditional lessons. This result confirms the study reported by Lopes and Ferreira (2024), in which those authors identified greater appeal and students' participation when proposing lessons with manipulable materials to teach mathematics.

Regarding students' participation and engagement, they were explored during the planning and application of activities in the classroom involving competition between groups. With that incentive, that is, investigation involving competition, more students participated, involving those that usually did not conclude the activities in the "traditional" mathematics lesson model, which Skovsmose (2000) calls exercise paradigm.

Therefore, the planning involving the Tower of Hanoi proposed that the first phase of the activity already presented a competition to call students attention to the remaining tasks, which demanded investigation and the material manipulation. From the students' interest, they could understand the game rationale and rules, which helped the development of the other challenges proposed involving manipulation in the investigation of patterns.

To enable mathematical investigation, some intervention by the scholarship holders was necessary at times to motivate the investigation and the "invitation acceptance". Therefore, regarding the challenge of the minimum number of movements, students were asked: "is this really the least number of movements?" or "how did you get this result?", and from that, it was possible to make them search for a kind of reasoning for any number of disks, and then they were asked "what happens if the number of disks is even?" "what if it is odd?", "is there a pattern?", "how would you write this reasoning using mathematics?", "is there a formula?", "which school content can you relate it with?". All these questions were part of the teachers' intervention (in this case the scholarship holders') so that the students investigated and got as close as possible to the expected result.

Some cases required greater intervention, which occurred in some classes in the identification of the minimum number of movements, in which one of the scholarship holders taught a game strategy relating the even number of pieces with the next movement to be made. However, this did not prevent the mathematical investigation since all groups accepted the invitation and students were engaged in the search for results. From the investigation carried out, many groups managed to reach a formula to find the least number of movements. On average, 100 students, that is, around 20 groups, were successful in finding the formula to describe the results obtained.

For the formalization of results, we thought it was important for the students to understand the proposal and see the task as more than an activity focusing on mathematics only. The reason for this is that in activities of this type, students commonly present difficulties in "translating" their thoughts into algebraic language. Therefore, even if students understand the proposal and explain the results found orally, it was necessary to present a formula describing the result obtained, since that was one part of the task proposed.

Aiming at keeping the mathematical investigation and optimizing the search for a formula, we questioned students about possible letters that they could use to represent the number of disks and their results. Students opted for using n for the number of disks and A_n for the result obtained. This helped them to "translate" their thought into algebraic language. From this experience, we thought it would be interesting if these students had lessons about algorithms, so that this algebraic thought could be explored in other problems. This is a suggestion for new approaches to this theme in the classroom.

After the scholarship holders' helped to organize each group's reasoning, the formulas were commonly related to the previous result, such as in the formula $2A_{n-1} + 1$, where *n* represents the number of disks and A_{n-1} the result obtained in the previous number of disks. The drawback of this formula is exactly the fact that you always need to know the previous result to be able to obtain a new one, which is a problem when you have a higher number of disks. However, with the scholarship holders' help and incentive, students managed to reach a general formula involving a number *n* of disks, which is $2^n - 1$.

Although most of the groups reached a formula representing the least number of movements, we could observe their difficulty in generalizing the mathematical thought and finding a pattern of disks. In addition, the fact that they tried to find the number from exploration also made the identification of a pattern more difficult since in most cases they found an incorrect number. For this reason, after students had tried to conclude this part of the challenge, the scholarship holders helped them with group exploration until they found the correct number.

Even with the scholarship holders' help, the difficulty students had in describing mathematically the reasoning was clear, and most times, the formula of the least number of movements was shared by one student or a group that stood out in helping the scholarship holders to build the formula. In general, the formula found was: $2(n-1) + 1$; and even reaching this formula, the scholarship holders presented the formula $2^n - 1$, which can also be related to the minimum number of movements.

This revealed another difficulty presented by the students, the potency content. In one of the scholarship holders' notes, a comment highlighted was "$2^3 = 12$", not only this, but in three classes, the scholarship holders had to go back to the potency content to help students understand why the formulas found were correct. This reinforces the importance of more activities being carried out with high school students, so that mathematical concepts that are widely used in higher education can be developed. In general, difficulties with basic mathematics tend to persist and are also identified in higher education, which might result in demotivation and failure in undergraduate courses, or even, in the professional activity of those students (Lopes; Queiroz, 2024).

The intervention played a significant role in the students' ability to establish links between the theory of geometric progressions and the challenge presented by the Tower of Hanoi game. This link between ludic practice and mathematical concepts contributed to the content consolidation, thus evidencing the efficacy of Skovsmose's (2000) approach. In addition, these results suggest that activities involving practical and ludic elements draw students' attention and favor the understanding and application of complex mathematical concepts. Therefore, the continuous and broad use of similar teaching practices is recommended aiming at promoting more appealing and effective learning at the high school level.

Since students in general tend to present difficulties with mathematics content, the exercises developed in the classroom can be observed from other angles. In addition to learning mathematics as a science, we could notice how mathematical education can be presented in a more significant, reflective, engaged, and socially active way as proposed by Skovsmose (2014). Since the activity was different from the traditional teaching pattern, it integrated students who were usually uninterested or found mathematics too difficult. It motivated them to solve the problem and interact socially making the learning more solid and significant.

As regards the teaching initiation scholarship holders' action, we highlight that the development of activities like the one presented here favors their learning and initial education, thus optimizing their teaching practice and recognition of the classroom as a space of future action. As future teachers of mathematics, the PIBID and the contact with the classroom are opportunities for personal development and getting familiar with mathematics teaching, new teaching methodologies, and the direct contact with teachers and students.

Final Considerations

The use of the Tower of Hanoi in a teaching intervention by means of a mathematical investigation showed to be an effective approach to improve the learning of mathematical content, more specifically progressions, in the high school context. Therefore, greater students' interest and participation were identified when compared to traditional activities, whose main characteristic is the repetition of exercises, that is, the exercise paradigm. In addition, the landscapes of investigation enabled the scholarship holders to explore an unusual task, thus increasing their repertoire of teaching methodologies and aligning theory and practice in the classroom.

At the same time, the PIBID presence in public schools has presented a relevant impact on teaching and learning since the participation of undergraduate students allows the exploration of current aspects of education and teaching methodologies. Another important aspect observed is the proximity of students since the activities enable greater interest in learning and some contact with the reality of undergraduate students.

We also consider the development of other activities based on landscapes of investigation and/or the use of manipulable materials important optimizers of significant learning of mathematical contents and as an alternative to the exercise paradigm of traditional models.

On future occasions, we hope undergraduate students along with their supervisor teachers and university professors carry out other activities with manipulable materials and approaches that involve mathematical investigations. A greater repertoire of activities to approach other mathematics content in the classroom will enable the investigation of and advancements in students' learning of mathematics with the use of different methodologies and tasks.

References

BRASIL. Ministério da Educação. **Base Nacional Comum Curricular.** Brasília, 2018.

CAPES. PIBID. 2023. Retrieved from: <https://www.gov.br/capes/pt-br/acesso-a-informacao/acoes-e-programas/educacao-basica/pibid/pibid>. Accessed on: 11 Jan 2024

COSTA, A. Torre de Hanói, uma proposta de atividade para o ensino médio. In: XVI Encontro Regional de Estudantes de Matemática do Sul. **Anais [EREMATSUL]**, Porto Alegre: EdiPUCRS, 2010. Retrieved from: https://editora.pucrs.br/edipucrs/acessolivre/anais/erematsul/comunicacoes/2ALEXANDREDACOSTA.pdf. Accesed on: 18 Aug 2023.

LOPES, R. A.; FERREIRA, J. C. O que pode o Triângulo de Sierpinski em salas de aula do Novo Ensino Médio? **SIGMAE**, v. 13, p. 1-21, 2024.

LOPES, R. A; QUEIROZ, C. R. O. Q. Influências e desdobramentos da monitoria acadêmica em disciplinas de Matemática em uma universidade federal: um olhar para as percepções dos estudantes. **REMAT: Revista Eletrônica da Matemática**, v. 10, p. e2001, 2024.

MINAS GERAIS. Secretaria de Estado da Educação. **Currículo Referência de Minas Gerais - CRMG.** Retrieved from: http://basenacionalcomum.mec.gov.br/images/implementacao/curriculos_estados/documento_curricular_mg.pdf. Accessed on: 04 Jan 2024.

PIBID - PROGRAMA INSTITUCIONAL DE BOLSA DE INICIAÇÃO À DOCÊNCIA. **gov.br**, 2013. Retrieved from: https://www.gov.br/capes/pt-br/acesso-a-informacao/acoes-e-programas/educacao-basica/pibid/pibid. Accessed on: 11 Jan 2024

SKOVSMOSE, O. Cenários para Investigação. v.13. Rio Claro: **Bolema,** 2000. [1]

SKOVSMOSE, O. **Um convite à educação matemática crítica.** Brasil: Papirus Editora, 2014.

This page was intentionally left blank – formatting consistency.