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Mathematical modeling of population dynamics in the city of Maringá, PR: parameter estimation through linear regression

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Abstract: *The aim of this study is to model the population growth of Maringá, Paraná, using classic Population Dynamics models and compare them with real data obtained from IBGE and datasus between 1980 and 2012. To achieve this, it is necessary to obtain the parameters of the models, which will be obtained through Linear Regression. After obtaining the parameters and comparing the results with real data, it was observed that the Verhulst model performed the best, with a small margin of error when compared to the actual population. Consequently, the Malthusian model proved impractical for population projection. Therefore, mathematical modeling has proven to be effective in simulating real-world problems, especially in population dynamics where understanding population behavior is crucial for future public policy creation and disaster prevention.*

Keywords: *Mathematical Modeling; Population Dynamics; Linear Regression; Maringá - PR.*

Modelagem matemática da dinâmica populacional na cidade de Maringá, PR: estimação de parâmetros por regressão linear

Resumo: *O objetivo deste trabalho é modelar o crescimento da população de Maringa-PR através de modelos clássicos de Dinâmica Populacional e comparar com os dados reais obtidos pelo IBGE e datasus entre os anos de 1980 e 2012. Para isso se faz necessário obter os parâmetros dos modelos, que serão obtidos através da Regressão Linear. Após a obtenção dos parâmetros e comparação dos resultados com os dados reais, observou-se que o modelo de Verhulst foi o que obteve melhores resultados, com uma pequena margem de erro quando comparado com a população real. Desta forma o modelo de Malthus ficou in- viável para uso da projeção da população. Dessa forma a modelagem matemática tem se mostrado eficaz na simulação de problemas reais, em específico em dinâmica populacional onde a investigação do comportamento da quantidade populacional é importante para a criação de políticas públicas futuras e prevenção de catástrofes.*

Palavras-chave: Modelagem Matemática; Dinâmica Populacional; Regressão Linear; Maringá – PR.

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Introduction

Population Biology is the study of biological populations, where a population is a group of individuals of the same species who have a high likelihood of interacting with each other. This concept is broader than what is considered in Population Ecology, as Population Biology includes aspects of Genetics and Evolution. To understand, explain, and predict the dynamics of biological populations, mathematical models are employed (Queiroz Dias, 2013).

Population dynamics explores the growth of populations, predicting their behavior. This type of study is essential because once we have an idea of the future behavior of a population, it becomes possible to plan public policies and make informed decisions.

Maringá has emerged as a thriving and prosperous city, with the economy that experienced the most significant growth among the three largest cities in Paraná in 2020. Even in a pandemic scenario, the Gross Domestic Product (GDP) of the city increased by 4%, surpassing R$ 20 billion. This growth was driven by the sectors of services, industry, public administration, and agriculture, reflecting the hard work of the people of Maringá, as well as the incentives promoted by the local government (CONEGERO, 2023). Therefore, the results of this research will be significant for the city of Maringá, Paraná.

Population Biology, a field that delves into the intricacies of how populations of organisms grow and evolve, holds valuable insights for urban planning and policymaking, especially in the context of a city like Maringá. By applying mathematical models and concepts from this field, we can gain a deeper understanding of the dynamics of human populations within the city.

As Maringá continues to grow economically and demographically, it becomes increasingly important to anticipate and plan for future population changes. This includes not only predicting population growth but also understanding how factors like genetics and evolution can influence the composition and characteristics of the population. Such insights can inform decisions related to infrastructure development, healthcare services, education, and environmental conservation.

Furthermore, the role of genetics and evolution in population biology is of relevance in the context of Maringá's economic and agricultural sectors. Understanding how genetic diversity within agricultural crops or livestock populations can impact food production and sustainability is crucial for informed policymaking and long-term planning.

In conclusion, the field of Population Biology, with its focus on mathematical modeling, genetics, and evolution, provides a valuable framework for comprehending and predicting the dynamics of biological populations, including human populations in urban settings like Maringá. By integrating these principles into urban planning and policymaking, Maringá can better prepare for the challenges and opportunities that lie ahead in its continued growth and development. This research has the potential to contribute significantly to the city's prosperity and well-being.

Mathematical modeling

Mathematical modeling plays a crucial role in various fields of human knowledge, offering a systematic and precise approach to understanding, analyzing, and solving a wide range of complex problems. As highlighted by Bassanezi (2014), this process involves the creation and validation of mathematical models, which play a fundamental role in solving specific issues, elucidating natural and social phenomena, and predicting future trends.

The essence of mathematical modeling lies in representing real-world problem situations through mathematical solutions. This requires a unique ability to translate the complexity of reality into equations and mathematical relationships that can be analyzed and manipulated to gain valuable insights. As Borges (2023) wisely observes, the ability to apply mathematics in various areas of knowledge lies in the capacity to transform intricate and multifaceted problems into well-defined mathematical models.

One of the fields where mathematical modeling finds significant application is the study of population growth. Since the early days of mathematics, thinkers have been interested in understanding and predicting how populations grow and develop over time. Two of the most notable classical models in this domain are the models of Malthus and Verhulst.

The Malthusian model, proposed by Thomas Malthus in the late 18th century, suggested that the population grows exponentially while the resources available to sustain that population increase only linearly. This discrepancy would inevitably lead to overpopulation crises unless birth control measures were implemented.

On the other hand, the Verhulst model, developed by Pierre Verhulst in the 19th century, introduced the idea of a population growth rate that decreases as the population approaches a limit known as carrying capacity. This results in a more realistic growth pattern where the population eventually stabilizes at a sustainable level, avoiding the catastrophe predicted by Malthus.

These classical models, although simplified, provided a fundamental basis for the study of population growth and continue to influence research in the fields of ecology, demography, and economics. However, the complexity of the real world often requires more elaborate models that consider additional factors such as migration, variable mortality rates, and limited resources.

In summary, mathematical modeling is a powerful and versatile tool that plays a vital role in problem-solving across various disciplines. It allows researchers and scientists to translate the complexity of the world into accessible equations, thus advancing our understanding and ability to predict future events. Population growth models, such as those of Malthus and Verhulst, are emblematic examples of this approach and remain relevant in the analysis of demographic and ecological issues.

Malthus

Malthus' model is a simple differential equation that can be used to make estimates for a small population, provided it has few or no natural constraints. This mathematical model aimed to calculate short-term demographic growth (10 to 20 years).

In his model, Malthus (1798) proposed a simple ordinary differential equation that can be solved using the method of separating variables. Thus, it is possible to develop significant estimates for the historical context in which the theory was developed.

In the population growth model without inhibition, the intrinsic growth rate r is considered constant and positive (BASSANEZI, 2014), meaning that the rate of change of the population with respect to time, denoted here as dP/dt, is proportional to the current population. In other words, $P=P(t)$ represents the population.

The Malthusian model is given by:

$$\frac{dP}{dt} = rP \quad (1)$$

where r is a proportionality constant (in this case, $r>0$). It is straightforward to verify that if $r>0$, we will have growth, and if $r<0$, we will have decay. This is a first-order linear ordinary differential equation (ODE), whose analytical solution is given by:

$$P(t) = P_0 e^{rt} \quad (2)$$

Where P0 is the initial population, that is, $P(0)-P_0$. Analyzing the parameter r leads to the following observations:

1. If $r>0$, the population grows and continues to expand towards ∞.
2. If $r<0$, the population will decrease and tend toward 0. In other words, the population will go extinct.

The first case, $r>0$, is not suitable, and the model may not work well in the long term due to the primary argument arising from environmental limitations.

The complication arises because population growth is eventually constrained by some factor, typically among those essential resources. When a population is far from its growth limit, it can grow exponentially, but when it is close to its limit, the population size can fluctuate.

Verhulst

The Verhulst model, also known as the logistic model, is a differential equation that describes the growth of a population $P(t)$ under certain conditions, considering a limit of available resources. The model proposed by Verhulst introduces a growth limit to the Malthusian model. Instead of the constant growth rate r, we have $h(P(t))=r-aP(t)$, where a is a positive constant. Let's consider $P=P(t)$ thus, we have

$$\frac{dP}{dt}=h(P)P \Rightarrow \frac{dP}{dt}=(r-aP)P$$

It is common to rewrite $a=r$, with K being the carrying capacity, the population's size limit. With this, we have the following ODE.

$$\frac{dP}{dt}=rP(1-\frac{P}{K})$$

According to Verhulst (1838), the equation considers both an intrinsic growth rate and an environmental carrying capacity, limiting population growth, and more accurately simulating the action of external factors on population growth. The graphical behavior of the logistic equation shows initial exponential growth followed by a deceleration, stabilizing around the carrying capacity value K. The analytical solution is given by the expression:

$$P(t)=\frac{P_0K}{P_0+(K-P_0)e^{-rt}} \quad (3)$$

This is the analytical solution of the Verhulst model, describing population growth limited by the environmental carrying capacity $K=r/a$, given an initial population P_0 with an intrinsic growth rate r.

Regressão Linear

In Figueiredo (2019), a method for obtaining parameters for the Malthus and Verhulst models to model the growth of the Brazilian population is presented. In this work, we will employ the same methodology to model the population of Maringá, Paraná. Linear regression approximates data from a table with n data points with a line of the form $y = ax + b$, where the values of a and b are given by:

$$a = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \quad (4)$$

and

$$b = \frac{n\sum_{i=1}^{n} Y_i \sum_{i=1}^{n} X_i^2 - \langle X, Y \rangle \sum_{i=1}^{n} X_i}{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \quad (5)$$

where $X = \{X_1, \ldots ,X_n\}$ and $Y = \{Y_1, \ldots ,Y_n\}$ are the data related to the variables x and y. Using the population data of Maringá-PR, we make Y represent the data of $\frac{1}{P}\frac{dP}{dt}$, and X represent the data of P. Therefore, the parameters of equations (2) and (3), are given by:

$$r = b \quad (6)$$

and

$$K = -\frac{r}{a} \quad (7)$$

Aplicação

For the application of the Malthus and Verhulst models, we will use population data for Maringá, Paraná, from the years 1980 to 2012, as obtained from datasus (2023). Using equations (4) and (5), we found the following values:

$$a = -9.92262841448748 \times 10^{(-8)} \quad (8)$$

$$b = 0.050897259136211 \quad (9)$$

In this way:

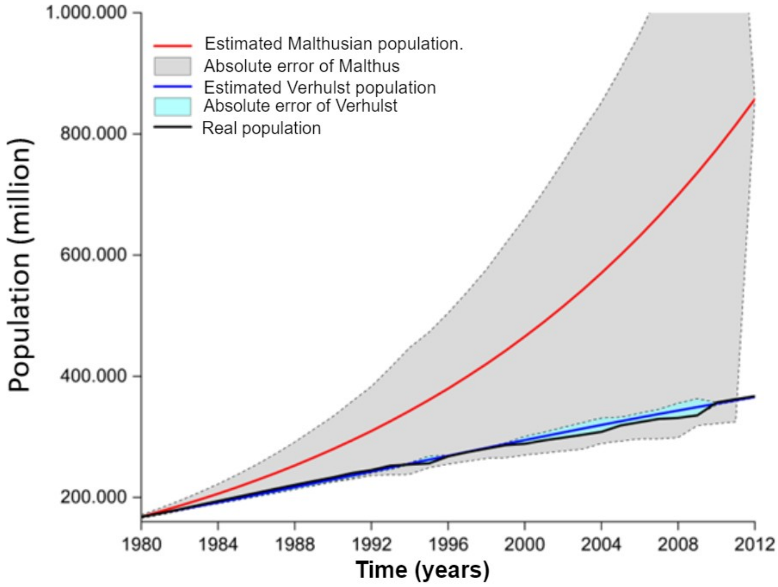
$$y = -9.92262841448748 \times 10^{-8} x + 0.050897259136211 \quad (10)$$

So, we have r and K

$$r = b = 0.050897259136211$$

$$k = -\frac{r}{a} = 512941$$

Figure 1: comparison of the Malthus and Verhulst models with real data.

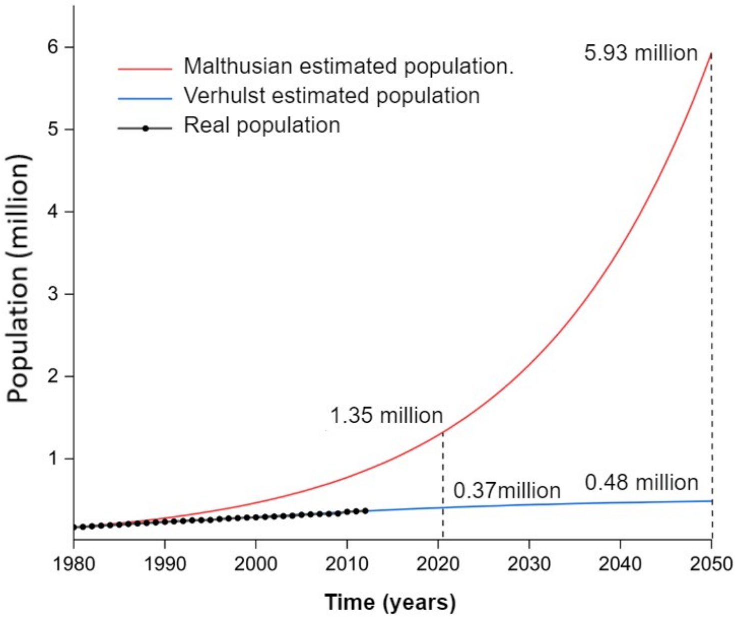


Fonte: Autores.

Figure 1 presents the outcomes obtained from the simulation of the Malthusian and Verhulst models, alongside a comparison with real-world data, along with the respective errors for each model. The superiority of the Verhulst model becomes evident when contrasted with the actual data, considering the observed errors. This advantage can be attributed to the nature of the Malthusian model, which represents exponential growth, resulting in a rapid increase in values.

Moving forward, Figure 2 provides a glimpse into the future population projection for Maringá, Paraná, employing both studied models. This projection offers valuable insights into the potential population trends and dynamics, aiding in informed decision-making and policy formulation for the city's sustainable development.

Figure 2: Future Projection of the Malthus and Verhulst Models for the city of Maringá, PR.



Fonte: Autores.

Conclusion

The results of this research clearly indicate the effectiveness of the Verhulst model in modeling the population dynamics of Maringá, Paraná, Brazil. This was achieved using linear regression to estimate the parameters necessary to calibrate the model. On the other hand, the Malthusian model was not suitable for predicting the city's population growth.

The application of linear regression to adjust the parameters of population dynamics models was a robust and reliable approach. It allowed for a precise adaptation of the models to the demographic reality of Maringá during the study period, which covered the years from 1980 to 2012.

The difference in performance between the Verhulst and Malthusian models underscores the importance of choosing the correct model to accurately predict population growth. The Malthusian model, characterized by exponential growth, did not align with the demographic characteristics of the city of Maringá, where population growth is moderated by various external factors.

Conversely, the Verhulst model, which considers a population carrying capacity, better fit the city's real data. This model recognizes the natural limitations of population growth and is more suitable for urban scenarios where resources are finite.

Furthermore, the integration of linear regression with population dynamics models has important practical implications, especially for urban planning and public policy formulation. This allows for a more accurate prediction of population evolution, assisting in informed decision-making and efficient resource allocation.

In summary, this research emphasizes the importance of the appropriate selection of population dynamics models and highlights the utility of linear regression in obtaining accurate parameters. This approach has significant implications for urban management and sustainable planning, as exemplified in the case of Maringá, Paraná, Brazil.

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